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Breakdown of the boundary-layer approximation for mixed convection above a horizontal plate

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Abstract—The mixed convection flow over a horizontal plate is investigated. Boundary-layer equations, modified to account for the hydrostatic pressure variation across the boundary layer, are solved numerically by a finite-difference scheme. If the (normal) fluid is cooled from below (or heated from above), the numerical results indicate that the derivative of the wall shear stress tends to infinity as a critical distance from the leading edge is approached. This occurs when the wall shear stress is till positive, i.e. before the classical separation criterion is satisfied. A mechanism of self-amplification is shown to be responsible for the breakdown.

1. INTRODUCTION

THE FORCED boundary-layer flow above a horizontal, heated or cooled plate is subject to a peculiar buoyancy effect. Obviously there is no buoyancy force tangential to the flat plate. However, since the temperature in the boundary layer differs from the ambient temperature, there is a hydrostatic pressure difference across the boundary layer. As the boundary-layer thickness increases, the hydrostatic pressure at the plate surface varies, too, with increasing distance from the leading edge. More specifically, in the boundary layer above a *heated* horizontal plate the density is less than the ambient density, provided that the thermal expansivity is positive (normal fluids). This gives rise to a decrease of the hydrostatic pressure at the surface with increasing distance from the leading edge, i.e. a favourable pressure gradient. On the other hand, above a *cooled* horizontal plate there is an adverse pressure gradient due to buoyancy.

Taking the hydrostatic pressure variation across the boundary layer into account requires an appropriate modification of the laminar boundary-layer equations. Following the works of Mori [1] as well as Sparrow and Minkowycz [2], several authors investigated the problem with the assumption of either constant plate temperature or constant surface heat flux. Perturbation series in terms of the distance from the leading edge apply only to small buoyancy effects [2–5] and are, therefore, subject to the same limitations as the results of second-order boundary-layer theory [6, 7]. On the other hand, the buoyancy-dominated case of a favourable pressure gradient and very large distances from the leading edge was considered in [5]. The limiting cases of very small and very large Prandtl numbers were studied by the method of matched asymptotic expansions [8]. Approximate solutions of the modified boundary-layer equations were obtained by an integral method similar to the Kármán–Pohlhausen method [9] as well as by local similarity and related methods [10, 11]. More recently, further results were provided by experimental investigations [12] and numerical solutions [13, 14]. Besides, a vortex instability of the fluid flow heated from below or cooled from above was found [23, 12].

Owing to these investigations [1–14] the effects of buoyancy on horizontal boundary-layer flow are now well documented in case of a favourable pressure gradient. As far as the case of an adverse pressure gradient due to buoyancy is concerned, however, previous investigations seem to be incomplete. While earlier attempts to extend the approximate solutions for constant wall temperature downstream to the point of vanishing shear stress failed [10], the problem of determining the distance from the leading edge where separation occurs was circumvented in the recent investigation [14], which provided velocity and temperature profiles but no shear stress and heat transfer distributions along the plate.

The difficulties in getting to the point of vanishing shear stress were attributed to convergence problems by some authors, e.g. [10, 14]. Other investigations, however, indicated that there might be a more fundamental problem involved. If the plate temperature varies proportionally to the inverse square root of the distance from the leading edge, an exact similarity solution can be found [15, 16] and interpreted to describe the mixed convection flow above a plate that is strongly heated or cooled at the leading edge while otherwise isolated, cf. [16] and, for a more general discussion, [19] and [20]. Based on the observation that the similarity solution does not exist if the adverse pressure gradient is larger than a certain critical value, which still yields positive shear stress, it was suggested in [16] that the boundary-layer approximation might also break down in the constant wall temperature (or constant heat flux) case before the point of vanishing shear stress is reached. More detailed investigations [17, 18] of the dual nature of the similarity solution near critical conditions provided further support for this point of view.

Therefore, the problem of mixed convection around

NOMENCLATURE

- a thermal diffusivity
- A, B coefficients depending on shapes of velocity and temperature profiles
- c_f coefficient of wall friction, $2\tau_w/\rho_\infty u_\infty^2$
- f dimensionless stream function, cf. equation (10)
- F $= u/u_\infty$, cf. also equation (14)
- g gravity constant
- Gr_x Grashof number, $g\beta T^* x^3/\nu^2$
- k thermal conductivity
- Nu_x Nusselt number, $q_w x/k(T_w - T_\infty)$
- p pressure
- Pr Prandtl number, ν/a
- q heat flux density
- Re_x Reynolds number, $u_\infty x/\nu$
- T temperature
- T^* characteristic temperature, cf. equation (9)
- u, v velocity components in x, y-directions (dimensionless in Section 5)
- x, y Cartesian co-ordinates in the plate direction and normal to it, respectively (dimensionless in Section 5).

Greek symbols

- β thermal expansivity
- δ boundary-layer thickness ($u/u_\infty = 0.999$)
- δ_2 momentum thickness
- ξ, η dimensionless independent variables, cf. equation (7)
- η_δ value of η where $u/u_\infty = 0.999$
- ϑ dimensionless temperature difference, $(T - T_\infty)/T^*$
- τ shear stress (dimensionless in Section 5)
- ν kinematic viscosity
- ρ density of the fluid
- ψ stream function.

Subscripts

- c critical value (breakdown)
- ∞ free-stream value (undisturbed fluid)
- w wall value (upper surface of plate).

a horizontal plate with constant wall temperature and constant heat flux, respectively, is re-examined in the following sections, with particular emphasis on the case of cooling the upper surface (or heating the lower one), i.e. of an adverse pressure gradient. The numerical results obtained by a finite-difference method show a singular behaviour that differs from the classical boundary-layer singularity near separation. An explanation for this peculiar break-down of the boundary-layer approximation is offered by considering the integral form of the momentum balance.

2. BOUNDARY-LAYER EQUATIONS

Consider the laminar boundary-layer flow on the upper side of a horizontal flat plate (Fig. 1). The free-stream values of velocity, temperature and density are denoted by u_∞, T_∞ and ρ_∞, respectively. Two different cases will be studied; (i) constant wall temperature, T_w; (ii) constant heat flux at the wall, q_w. Constant fluid properties are assumed, and the Boussinesq approximation is applied.

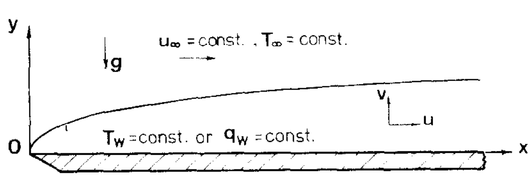

FIG. 1. Mixed convection boundary-layer flow over a horizontal plate: co-ordinate system and boundary conditions.

The governing equations can be written in Cartesian co-ordinates x, y as follows (cf. [2, 10, 16]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_\infty}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{1}{\rho_\infty}\frac{\partial p}{\partial y} = g\beta(T - T_\infty), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where u and v are the velocity components in the x- and y-directions, respectively, T is the temperature of the fluid, p the pressure, ν the kinematic viscosity, β the (positive) thermal expansivity and a the thermal diffusivity. According to the boundary-layer assumption, the momentum equation in the y direction is reduced to equation (3), which accounts for the hydrostatic pressure gradient.

Integrating (3) with respect to y, and substituting for p in equation (2) yields

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\frac{\partial}{\partial x}\int_y^\infty (T - T_\infty)\,\mathrm{d}y. \tag{5}$$

Equations (1), (4) and (5) form a system of equations for the three unknowns u, v and T. This system has to satisfy the following boundary conditions.

(i) For the case of constant wall temperature:

$$\begin{aligned} u = v = 0,\quad T &= T_w \quad \text{at } y = 0; \\ u = u_\infty,\quad T &= T_\infty \quad \text{as } y \to \infty. \end{aligned} \tag{6a}$$

(ii) For the case of constant heat flux at the wall:

$$u = v = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad \text{at } y = 0; \qquad u = u_\infty, \quad T = T_\infty \quad \text{as } y \to \infty; \tag{6b}$$

where $q_w > 0$ for a heated plate while $q_w < 0$ for a cooled plate.

To express the governing equations in dimensionless form, one introduces new independent variables ξ, η [10] as follows:

$$\xi = \frac{g\beta T^*}{u_\infty^2}\left(\frac{vx}{u_\infty}\right)^{1/2} = \frac{Gr_x}{Re_x^{5/2}}, \quad \eta = y\left(\frac{u_\infty}{vx}\right)^{1/2} \tag{7}$$

with

$$Gr_x = \frac{g\beta T^* x^3}{v^2}, \quad Re_x = \frac{u_\infty x}{v} \tag{8}$$

and

$$\left.\begin{aligned} T^* &= T_w - T_\infty \quad &\text{if } T_w = \text{const};\\ T^* &= \pm u_\infty\left(\frac{|q_w|}{g\beta k}\right)^{1/2} \quad &\text{if } q_w = \text{const.}\end{aligned}\right\} \tag{9}$$

The minus sign in the latter equation is associated with the case of the cooled wall. Note that $T^* < 0$, $Gr_x < 0$ and $\xi < 0$ for the cooled plate.

Furthermore, a dimensionless stream function and a dimensionless temperature are defined according to the relations

$$f(\xi,\eta) = (u_\infty vx)^{-1/2}\psi(x,y), \qquad \vartheta(\xi,\eta) = (T-T_\infty)/T^* \tag{10}$$

where $\psi(x,y)$ is the stream function, which is introduced to satisfy the continuity equation (1). Substitution of equations (7)–(10) into equations (4)–(6) leads to the following dimensionless form of the momentum and energy equations (primes and subscripts ξ denoting differentiation with respect to η and ξ, respectively):

$$2f''' + ff'' = \xi(f'f'_\xi - f''f_\xi) - \xi\left[\eta\vartheta + \xi\int_\eta^\infty \vartheta_\xi \, d\eta + \int_\eta^\infty \vartheta \, d\eta\right]; \tag{11}$$

$$2Pr^{-1}\vartheta'' + f\vartheta' = \xi(f'\vartheta_\xi - f_\xi\vartheta'). \tag{12}$$

The appropriate boundary conditions are, in the case of constant wall temperature:

$$f'(\xi,0) = f(\xi,0) = 0, \quad f'(\xi,\infty) = 1; \qquad \vartheta(\xi,0) = 1, \quad \vartheta(\xi,\infty) = 0; \tag{13a}$$

and in case of constant heat flux at the wall:

$$f'(\xi,0) = f(\xi,0) = 0, \quad f'(\xi,\infty) = 1, \qquad \vartheta'(\xi,0) = -|\xi|, \quad \vartheta(\xi,\infty) = 0. \tag{13b}$$

Numerical techniques are more conveniently applicable if equation (11) is rewritten as a second-order partial differential equation by introducing $F\,(=u/u_\infty)$ as a new dependent variable as follows:

$$f(\xi,\eta) = \int_0^\eta F(\xi,\eta)\,d\eta. \tag{14}$$

Thus equation (11) becomes

$$2F'' + fF' = \xi(FF_\xi - f_\xi F') - \xi\left[\eta\vartheta + \xi\int_\eta^\infty \vartheta_\xi\,d\eta + \int_\eta^\infty \vartheta\,d\eta\right]. \tag{15}$$

3. NUMERICAL PROCEDURE

The system of equations (12), (14) and (15) subject to the boundary conditions (13a) or (13b) was solved numerically by the finite-difference method following Patankar and Spalding [21]. At each step in the ξ-direction, the integrals in (15) were calculated by Simpson's method based on the values of ϑ_ξ and ϑ at the preceding position of ξ. The initial profiles of f and ϑ at $\xi = 0$ were obtained from (11) and (12), which, for $\xi = 0$, reduce to a system of ordinary differential equations with appropriate boundary conditions following from equation (13a) or (13b). While a constant step size $\Delta\eta$ appeared to be sufficient in the lateral direction, sufficiently accurate results close to the point of breakdown could be obtained only with variable step size in longitudinal direction; i.e. $\Delta\xi$ was taken to be inversely proportional to $\Delta f''_w$, where $\Delta f''_w$ is the difference of the second derivative of the dimensionless stream function at the wall between two successive lines $\xi = \text{const}$.

In order to check the accuracy, the calculations were repeated with various values of $\Delta\eta$ and $(\Delta\xi)_i$ (i.e. initial value of $\Delta\xi$). For the results to be presented, $\Delta\eta = 5\times10^{-2}$ and $(\Delta\xi)_i = 5\times10^{-3}$ and 1×10^{-3} for, respectively, heated and cooled plates (favourable and adverse pressure gradients) were found to be sufficient. More details of the numerical procedure, including the difference scheme and the set of difference equations to be solved, are given in [22].

When the velocity and temperature fields have been obtained, the local Nusselt number Nu_x and the friction coefficient c_f can be determined. Defining

$$Nu_x = q_w x/k(T_w - T_\infty), \quad c_f = 2\tau_w/\rho_\infty u_\infty^2, \tag{16}$$

with $q_w = -k(\partial T/\partial y)_{y=0}$ and $\tau_w = \rho_\infty v(\partial u/\partial y)_{y=0}$ and introducing dimensionless variables, we obtain

$$\tfrac{1}{2}c_f Re_x^{1/2} = f''(\xi,0); \tag{17}$$

$$Nu_x Re_x^{-1/2} = -\vartheta'(\xi,0) \quad \text{if } T_w = \text{const}; \tag{18a}$$

$$Nu_x Re_x^{-1/2} = |\xi|/\vartheta(\xi,0) \quad \text{if } q_w = \text{const}. \tag{18b}$$

4. NUMERICAL RESULTS

The correctness of the present numerical procedure and the computer program has been checked by comparison with the results of previous investigations, cf. Figs. 2(a) and (b). Further numerical results are presented in Figs. 3(a–c) (constant wall temperature)

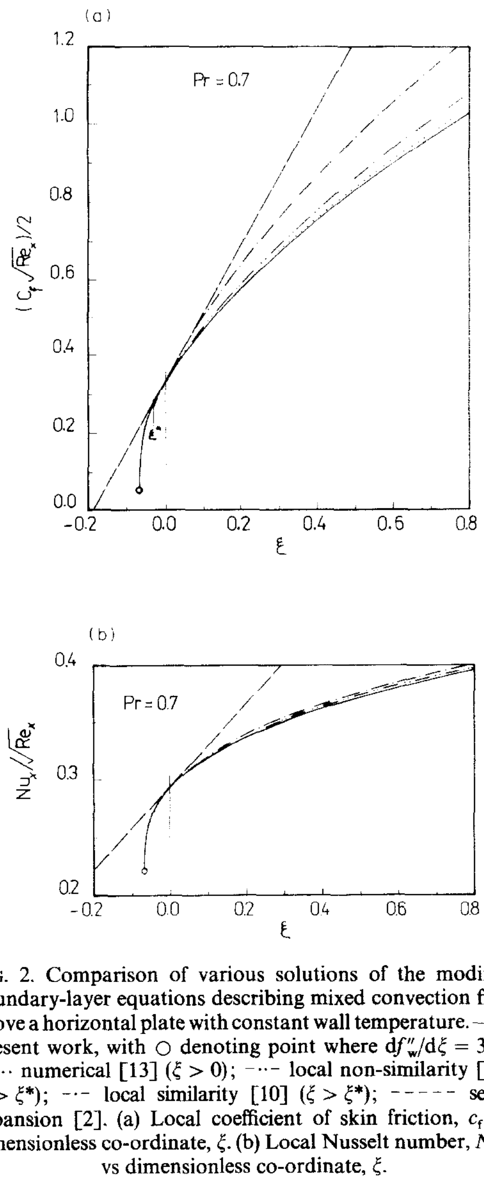

FIG. 2. Comparison of various solutions of the modified boundary-layer equations describing mixed convection flow above a horizontal plate with constant wall temperature. —— Present work, with ○ denoting point where $df''_w/d\xi = 300$; ...... numerical [13] ($\xi > 0$); –··– local non-similarity [10] ($\xi > \xi^*$); –·– local similarity [10] ($\xi > \xi^*$); ----- series expansion [2]. (a) Local coefficient of skin friction, c_f, vs dimensionless co-ordinate, ξ. (b) Local Nusselt number, Nu_x, vs dimensionless co-ordinate, ξ.

and 4(a) and (b) (constant wall heat flux). While the results for favourable pressure gradients ($\xi > 0$) show good agreement with previous data as far as the latter are available, the adverse pressure gradient ($\xi < 0$) is shown to cause a peculiar breakdown similar to the one found in the exact similarity solutions [16]. It is seen from Figs. 3(a) and (b) and 4(a) and (b) that very strong variations in skin friction and heat transfer appear when a certain critical distance from the leading edge is approached. This indicates a breakdown of the boundary-layer approximation. The critical values of the dimensionless longitudinal co-ordinate ξ are given in Table 1 for various Prandtl numbers. Note that the breakdown occurs at a finite, positive value of the wall shear stress, i.e. before the classical separation criterion of vanishing wall shear stress is satisfied. The velocity profiles near the point of breakdown confirm these findings, cf. Fig. 3(c).

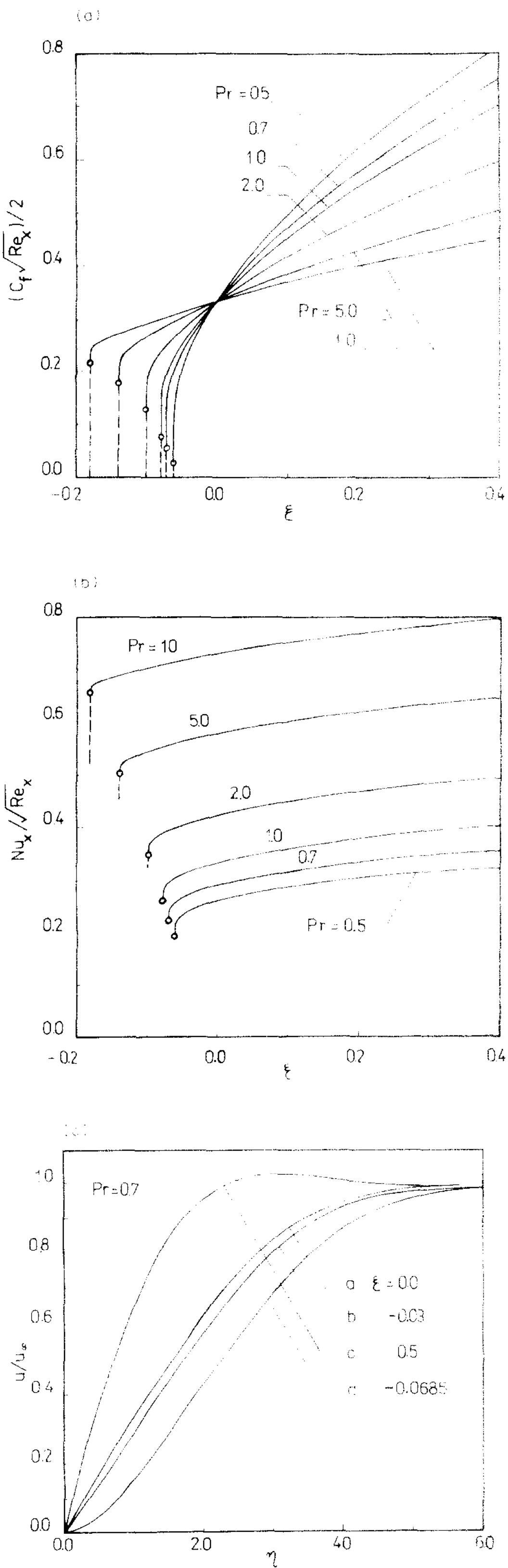

FIG. 3. Mixed convection flow above a horizontal plate with constant wall temperature; heated ($\xi > 0$) and cooled ($\xi < 0$) plate. (a) Local coefficient of skin friction, c_f. (b) Local Nusselt number, Nu_x. (c) Velocity profiles.

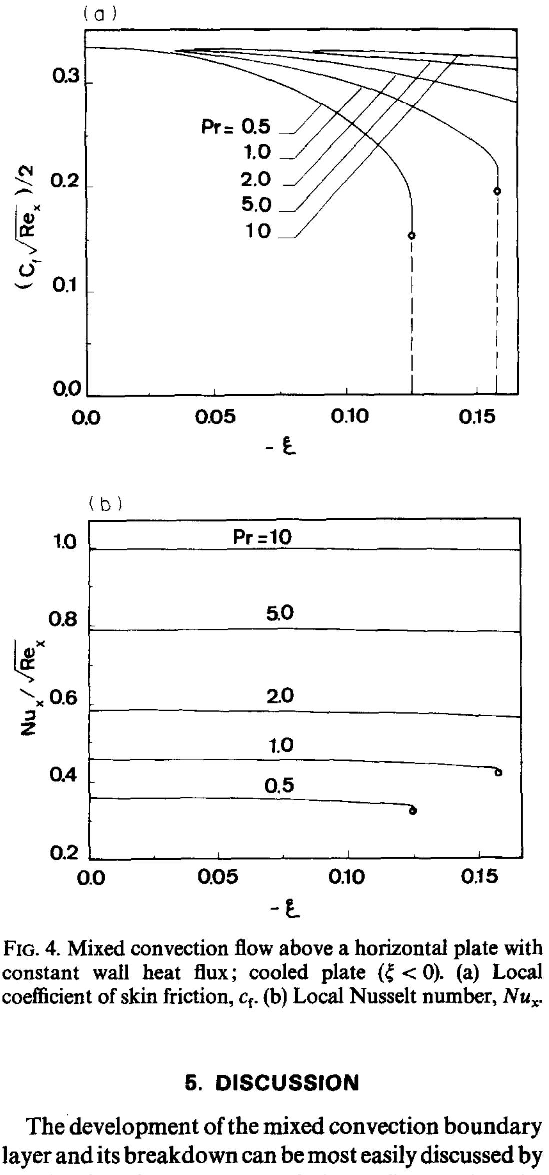

FIG. 4. Mixed convection flow above a horizontal plate with constant wall heat flux; cooled plate ($\xi < 0$). (a) Local coefficient of skin friction, c_f. (b) Local Nusselt number, Nu_x.

5. DISCUSSION

The development of the mixed convection boundary layer and its breakdown can be most easily discussed by considering the momentum balance. Integrating the momentum equation (5) with respect to y, and making use of the continuity equation (1), yields

$$\frac{d}{dx}\int_0^\infty \left[u(1-u) \pm \int_y^\infty \vartheta \, d\bar{y}\right] dy = \tau_w, \qquad (19)$$

where all quantities have been made dimensionless by referring the velocity u to the free-stream velocity u_∞, the wall shear stress to $\rho_\infty u_\infty^2$, and the x and y co-ordinates to $u_\infty^5/(g\beta T^*)^2 \nu$ and $u_\infty^2/g\beta|T^*|$, respectively. Upper and lower signs in (19) refer to the heated and cooled (upper) surface, respectively, and the dimensionless temperature disturbance ϑ has been introduced according to equation (10). For the purpose of comparison with the results of the numerical computations note that $x = \xi^2$, $y = |\xi|\eta$.

Introducing the momentum thickness

$$\delta_2 = \int_0^\infty u(1-u)\, dy \qquad (20)$$

the momentum balance (19) can be re-written as

$$\frac{d}{dx}(\delta_2 \pm A\delta_2^2) = B\delta_2^{-1} \qquad (21)$$

with the coefficients A and B,

$$A = \delta_2^{-2}\int_0^\infty \int_y^\infty \vartheta \, d\bar{y}\, dy, \qquad (22a)$$

$$B = \delta_2 \left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad (22b)$$

depending only on the shapes of the velocity and temperature profiles. For instance, sinusoidal profiles together with the assumption that velocity and temperature boundary layers are approximately of equal thickness if $Pr \approx 1$, yield $A = 2(\pi^2-8)/(4-\pi)^2 = 5.074$, $B = 1-(\pi/4) = 0.2146$.

For a qualitative discussion, A and B are assumed to be constant, which permits equation (21) to be integrated with the result

$$\delta_2^2(3 \pm 4A\delta_2) = 6Bx, \qquad (23)$$

where a constant of integration has been omitted to satisfy the initial condition $\delta_2 = 0$ at $x = 0$. When the plate (upper surface) is cooled, the minus sign in (23) applies, and the solution becomes singular, i.e. $d\delta_2/dx \to \infty$, as the 'critical' point $\delta_2 = \delta_{2,c}$, $x = x_c$ is approached, with

$$\delta_{2,c} = (2A)^{-1}, \; x_c = (24A^2B)^{-1}. \qquad (24)$$

Table 1. Critical values of ξ where boundary-layer breakdown is predicted, and skin friction coefficients at points where $df_w''/d\xi = 300$.

	Pr	0.5	0.7	1.0	2.0	5.0	10.0
Constant wall temperature	$-\xi_c$	0.0610	0.0685	0.0775	0.0991	0.1389	0.1800
	$\frac{1}{2}c_f Re_x^{1/2}$	0.0260	0.0530	0.0736	0.1271	0.1792	0.2153
Constant wall heat flux	$-\xi_c$	0.1247		0.1571	0.1967		
	$\frac{1}{2}c_f Re_x^{1/2}$	0.1515		0.1938	0.2267		

Furthermore, τ_w remains finite at the critical point, while $d\tau_w/dx$ grows beyond bounds as follows:

$$\tau \to \tau_{w,c} = 2AB,$$

$$\frac{d\tau_w}{dx} \to -\frac{\tau_{w,c}^2}{\delta_{2,c}-\delta_2} \to -\infty \quad \text{as } x \to x_c. \qquad (25)$$

With the A and B values given above, one obtains $x_c = 7.54\times10^{-3}$ and $\xi_c = -0.087$. Given the crude approximations that have been made, this result is sufficiently close to the numerical result for $Pr = 1$, cf. Table 1.

The analysis presented above suggests that the following mechanism is responsible for the boundary-layer breakdown observed in the numerical investigations. As the boundary-layer thickness increases, the adverse hydrostatic pressure increases too, which gives rise to a deceleration of the flow and, consequently, to a further increase of the boundary-layer thickness. This self-amplification eventually leads to an infinite growth rate of the boundary-layer thickness and of other flow quantities. Unlike the well-known singular behaviour of a boundary layer near the point of separation in a prescribed pressure gradient, the breakdown induced by the hydrostatic pressure gradient is not related with vanishing wall shear stress.

6. CONCLUSIONS

Numerical investigations supported by an approximate analysis indicate that the boundary-layer flow of a normal fluid (positive thermal expansivity) above a cooled horizontal plate (adverse pressure gradient) has a peculiar singularity. As a critical distance from the leading edge is approached, the derivative of the wall shear stress becomes infinite, while the wall shear stress itself remains finite and non-zero. Thus the boundary-layer approximation breaks down before the classical separation criterion of vanishing wall shear stress is satisfied.

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LIMITATION DE L'APPROXIMATION DE COUCHE LIMITE POUR LA CONVECTION MIXTE SUR UNE PLAQUE HORIZONTALE

Résumé—On étudie l'écoulement de convection mixte sur une plaque horizontale. On résout par une méthode de différences finies les équations de couche limite, modifiées pour tenir compte de la variation de pression hydrostatique à travers la couche limite. Si le fluide (normal) est refroidi par le bas (ou chauffé par dessus), les résultats numériques montrent que la dérivée du cisaillement pariétal tend vers l'infini quand on approche une distance critique du bord d'attaque. Ceci se réalise quand le cisaillement pariétal demeure positif, c'est-à-dire avant que le critère de séparation n'aparaisse. Un mécanisme d'auto-amplification est responsable de cela.

VERSAGEN DER GRENZSCHICHTNÄHERUNG FÜR GEMISCHTE KONVEKTION AN EINER HORIZONTALEN PLATTE

Zusammenfassung—Die gemischte Konvektionsströmung an einer horizontalen Platte wird untersucht. Modifizierte Grenzschichtgleichungen, in denen die hydrostatische Druckänderung quer zur Grenzschicht berücksichtigt ist, werden mit einem Differenzenverfahren numerisch gelöst. Für eine von unten gekühlte oder von oben beheizte, normale Flüssigkeit liefern die numerischen Ergebnisse ein unbegrenztes Anwachsen der Ableitung der Wandschubspannung, wenn man sich einer kritischen Entfernung von der Vorderkante nähert. Dabei ist die Wandschubspannung noch positiv, das heißt, die klassische Ablösebedingung ist noch nicht erfüllt. Als Ursache für das Versagen der Grenzschichtnäherung wird ein Mechanismus der Selbst-Verstärkung angegeben.

ГРАНИЦЫ ПРИМЕНИМОСТИ ПРИБЛИЖЕНИЯ ПОГРАНИЧНОГО СЛОЯ ДЛЯ СМЕШАННОЙ КОНВЕКЦИИ НАД ГОРИЗОНТАЛЬНОЙ ПЛАСТИНОЙ

Аннотация—Исследуется смешанная конвекция над горизонтальной пластиной. Уравнения пограничного слоя, модифицированные с целью учета изменения гидростатического давления поперек пограничного слоя, решаются численным методом конечных разностей. В случае охлаждения ньютоновской жидкости снизу (или нагрева сверху) численные результаты показывают, что при приближении к критическому расстоянию, отсчитываемому от передней кромки, производная от промежуточными значениями продольного шага. Для труб второго ряда Нуссельта для шахматных пучков обычно превышают соответствующие числа для решеток с расположением труб друг за другом, а также менее чувствительны к величине продольного шага. Суммарные перепады давления для двухрядных шахматных решеток также были практически независимы от продольного шага, причем их значения вдвойне превышали соответствующие значения для однорядной решетки. С другой стороны перепады давления для решеток с коридорным расположением труб были обычно вдвойне меньше, чем соответствующие значения для однорядной решетки и имели тенденцию роста с увеличением продольного шага.